

# NONPERTURBATIVE INSTABILITY OF BLACK HOLES IN QUANTUM GRAVITY

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## Abstract

The final stage of a black hole evaporation due to the Hawking effect is studied. One finds that, including the effects of quantum gravity, a black hole does not evaporate completely losing its energy steadily to a flux of created particles, but rather decays via a change in topology into an asymptotically flat space and an object which is a closed Friedmann Universe. This process is a genuine non-perturbative effect of quantum gravity and becomes the dominant “channel” of a black hole decay for black holes with masses slightly larger than the Planck mass  $M_p = 10^{19}\text{GeV}$ . We calculate the decay rate of a Schwarzschild black hole with the mass  $M$  and discuss other decay “channels” by topology change. An explicit instanton mediating the decay is constructed by matching the Schwarzschild and the “wormhole” Friedmann instantons on the minimal sphere which is a Euclidean section of the event horizon. We show, as an example, that the decay process is

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mediated in the semiclassical approximation by the gravitational-axionic instanton. However, we argue that the phenomenon discussed in this Letter does not depend on the particular instanton approximation and should be discussed in the framework of the second quantization of interacting geometry suggested in Ref. 18. It is argued that in the more general setting of the Wheeler-De Witt equation the wave functional describing a black hole is not gaussian because of the existence of an unstable mode.

Several years ago Hawking discovered that black holes are quantum mechanically unstable [1]. In the strong gravitational field of a black hole, the in- and out-vacua of different matter fields are different even if the object is characterized by the stationary gravitational field. It is the presence of the event horizon, whose sections are topologically  $S^2$ , that makes the difference. The nontrivial topology of black holes and their highly nontrivial quantum mechanical properties is probably the strongest argument in favor of considering quantum fluctuations in the topology of spacetime. The general concept of quantum fluctuations in the metric (the perturbative sector) and a possible topology change with the “pinching off” of separate Universes (or regions of spacetime) was first discussed by Wheeler [2], and more recently by Hawking [4,6]. Zel’dovich [3,5] has discussed the role of topology change and its implications for particle physics and cosmology. He also suggested that small black holes with masses close to the Planck mass  $M_p$  would decay in one quantum jump and a small closed world will be formed in such a process [5] together with an asymptotically Minkowski spacetime. Zel’dovich envisaged that such a process would necessarily violate the baryon and lepton number conservation law. Hajicek [16] in series of penetrating and very beautiful papers addressed the issue of the quantum mechanics of spin-zero black holes. He and his collaborators quantized the spherically symmetric gravitational and matter fields in the

topological black hole sector. Hajicek was able to solve the hamiltonian constraints and derived the reduced highly nonlinear but positive hamiltonian for the solitonic (extremal) black holes and matter fields. Indeed, this is the only example known to me where the serious attempt was made to quantize black holes and matter fields beyond the semiclassical approximation.

The physical implications of topology of black holes were recently discussed in the context of string theory [7]. In particular the fact that the second homology of a black hole manifold is nontrivial implies the existence of string world-sheet instantons whose role is to destabilize a black hole. The general picture which emerged from this work is that very small black holes as described by string theory undergo a sort of the phase transition to a different phase. When the Hawking temperature becomes large and comparable to the Hagedorn temperature, the string partition function at genera zero and one diverges, signaling instability. The effects of string world-sheet instantons and vortices lead to instability at genus zero. The effects of the Wess-Zumino-Novikov-Witten term (the two-form  $B$  field) on a black hole dynamics in string theory were discussed and a critical value for the mass of a black hole was found [7].

Indeed, a very small black hole of mass  $M$  comparable to the Planck mass  $M_p$  has a very high Hawking temperature  $T = M_p^2/8\pi M$ . The average thermal energy of the particles emitted by such black holes is slightly below the Planck mass, and it is definitely favorable for a black hole to “disappear” in a quantum fluctuation (as a discrete “quantum jump”), with a possible change of topology [5]. Also the average number  $N$  of particles produced in the black hole decay is of order  $N \cong 8\pi M^2/M_p^2$ . Notice that this number is proportional to the geometrical scattering cross-section or the area of a black hole horizon. Imagine now that a black hole is formed as an intermediate state in the collision of high energy elementary particles, say baryons. Such a state would decay rapidly producing a number of other particles. We do not know the hamiltonian describing such a process, because we do not have yet the microscopic fundamental theory of all interactions. On the other hand we know that the multi-particle production in

the high energy collision of hadrons can be phenomenologically described by the thermodynamical models. Such an approach to hadron collisions proved to be moderately succesful even if the average number of produced particles was not very large. One would expect that such an approximation becomes more accurate as the number of produced particles increases. The metastable intermediate state tends to behave thermodynamically as the number of “fragments” increases, which indicates that, even if the quantum coherence is not lost in such a process, the phase space becomes very large. In the fundamental theory of all interactions quantum black holes would probably appear as such “resonances”, or collective excitations, which eventually would “fragment” into a number of particles. The fact that the quantum mechanical decay of such a state can be described by thermodynamics simply reflects the hierarchy problem in quantum gravity (QG). Indeed, the mass scale of QG  $M_p$  is much above the energy scale of other fundamental interactions. It seems that this property of QG might be responsible for the large number of particles produced in the decay of a quantum black hole.

The nonperturbative instability of black holes by topology change will become the dominant channel of decay for very small Planckian black holes. The usual Hawking effect decay mode of particle creation probably becomes nondominant as the genuine quantum gravity (QG) effects start to dominate. The standard picture of final stages of the black hole evaporation does not seem to take into account the quantum effects of gravity. In particular, it seems unphysical to suggest that a black hole evaporates completely and that the resulting spacetime will possess a “naked singularity”. The final stage of a black hole evaporation due to the Hawking effect is a unique laboratory for any theory of quantum gravity. The Einstein-Hilbert action may not be a very good approximation of QG at the Planck energy scale  $M_p$ , but presumably one may consider it as the leading term in the gradient expansion of the low energy effective action for energy scales slightly below the Planck energy.

This assumption validates the use of the Wheeler-De Witt (WDW) equation [9] for the description of small “quantum black holes”. Consider the real time, i.e., Lorentzian, configuration of the gravitational field, say a black hole. In the

quantum mechanics of the gravitational field [9-12], one associates a wave function to this configuration. It is known how to do this, at least semiclassically, in a WKB approximation. One considers a Riemannian three-manifold  $\Sigma$  which is an initial data hypersurface  $\Sigma$  in classical general relativity (GR). The initial data is a canonical pair  $(h_{ij}, \pi_{ij})$ , a “point” in the phase space where the semiclassical wave function is localized. This assumption is based on the analogy with quantum mechanics of simpler systems. Usually one chooses the polarization on the classical phase space such that the WDW wave function is a functional of the three-metric  $h_{ij} : \Psi[h_{ij}]$ .

By the choice of the classical solution or, equivalently, by choosing a specific initial data on  $\Sigma$ , one associates with it the semiclassical wave function in the classically allowed region of a phase space. Now a given classical solution is a critical point of the action. By expanding the action around a classical solution one finds in the functional integral approach the gaussian wave function of a “ground state” [11,15,20]. The other way to do it is to solve the WDW wave equation and find the  $O(\hbar^0)$  quantum correction to the WKB wave function.

A “ground state” wave function may cease to be gaussian in some directions in the configuration space [15,20]. This behavior signals instability. Such a “ground state” is a “false vacuum”. It tends to decay, i.e., there exists an “overlap” between the “false vacuum” and another “vacuum”. The wave function tunnels to a classically forbidden region. Such a tunnelling is most conveniently described in the path integral approach where one can calculate the transition amplitude in a WKB approximation. This leads automatically to an instanton mediating such a transition. In this Letter we are interested in studying the semiclassical nonperturbative instability of “small” black holes.

In general relativity a black hole is stable. The topology of  $\Sigma$  does not change in classical general relativity because it would lead to causality violation. Therefore, the system under consideration tunnels to another classically allowed region of the phase space if a semiclassical instability is really present. In general relativity (GR)

the tunnelling can be associated with topology change [8,9,20]. This can be seen heuristically in the Feynman sum over histories approach where one sums over all possible “histories” with the amplitude  $\exp(iS[g])$  as the weight factor, where  $S[g]$  is the classical action. The “history” corresponding to topology change does not correspond to a solution of the classical Einstein equation (here we define the solution as the Lorentzian geometry with the smooth light cone structure, at least outside event horizons). As a result the action will not have an extremum, and in the sum over histories the total amplitude will be exponentially small because of the effects of destructive interference. This leads to the Euclidean continuation of the action in the semiclassical approximation and to the Euclidean QG prescription. For our purposes in this Letter we simply assume that the standard Euclidean arguments are applicable (this may not be true beyond the tree level).

Studying the semiclassical instability of black holes due to topology change in QG, one must find an instanton which mediates this instability. The classical “ground state” of a black hole is uniquely described by the Schwarzschild solution, which has the topology  $R^2 \times S^2$ . The presence of an instability can be established most easily by analytically continuing this solution to a Euclidean space signature, so that the metric is

$$ds^2 = (1 - 2GMr^{-1})d\tau^2 + (1 - 2GMr^{-1})^{-1} + r^2 d\Omega^2, \quad (1)$$

where  $M$  is a black hole mass and  $G = M_p^{-2}$  is the Newton constant given in terms of the Planck mass  $M_p$ . The Schwarzschild radius is  $R = 2MM_p^{-2}$ . The constant time section of this geometry has topology  $R \times S^2$ . This is the famous Einstein-Rosen bridge or the three-dimensional wormhole connecting two asymptotically flat regions. If one restricted the range of the radial coordinate to  $R < r < \infty$ , this three-geometry would be an incomplete manifold. However, one can see that the origin of this incompleteness is the presence of a “hole” at  $r = R$ . This “hole” is a minimal  $S^2$  on the manifold and one can double the manifold by gluing in the exact copy of it at the minimal  $S^2$ . In this way one obtains the Einstein-Rosen bridge.

This procedure corresponds to the maximal analytic extension of the Schwarzschild solution and is not dictated by the laws of physics. Evidently the realistic black hole formed in a gravitational collapse is asymmetric in this respect. Quantum mechanically a black hole radiates particles, out of the vacuum, which escape to the asymptotic region. Now this positive energy flux of outgoing particles is always accompanied by a negative energy flux crossing an event horizon. The situation is time asymmetric. Therefore, physically the initial constant time section is only half of the Einstein-Rosen bridge. One can find a coordinate system on half of the wormhole such that the metric is conformal to the metric on half of the three-sphere  $S^3$ . However, the conformal factor will be singular at one point, this corresponding to the fact that the topology of the bridge is different than a three-dimensional disk  $D^3$ . The singular point on this disk is the “infinity” on the bridge. One sees that by compactifying the bridge by adding a point at infinity, or equivalently by conformally rescaling the metric, we obtain a three-manifold which is topologically  $S^3$ , with the boundary  $S^2$  being a minimal two-sphere.

How do we search for a semiclassical instability of some given configuration? The proper tool we need to address the question of topology change is cobordism theory. A general discussion of this approach was presented in Ref.(9) . Let  $\Sigma$  be an initial three-geometry and  $\Sigma'$  be a final three-geometry. The question of semiclassical instability of  $\Sigma$  can be formulated now as the problem of the existence of a smooth Riemannian manifold  $\mathcal{M}$  interpolating between  $\Sigma$  and  $\Sigma'$ . Two oriented manifolds are called cobordant if their disjoint union bounds a smooth manifold,  $\partial\mathcal{M} = \Sigma \cup \Sigma'$ . The basic result of cobordism theory which is useful here is that two manifolds are cobordant if their Pontryagin and Stiefel-Whitney numbers are equal. This implies in particular that all closed oriented three-manifolds are cobordant. This also means that  $S^3$  can “decay” into any closed arbitrarily complicated oriented three-manifold. If a manifold is compact and has a boundary then one can modify the present argument and consider cobordisms with fixed boundaries.

Now a three-sphere with a boundary  $S^2$  is cobordant to any oriented three-manifold with  $S^2$  boundary. This means that, at least in principle, a black hole

can decay into any topologically nontrivial configuration. Consider, as in Figure 1, the complete three-manifold obtained by “filling in” a hole in  $R^3$ , i.e., by gluing in a disk  $D^3$  (or  $S^3$  with a boundary  $S^2$ ) to half of the Einstein-Rosen bridge:  $R \times S^2 + D^3 = R^3$ . Adding a sphere  $S^3$  to a disk  $D^3$  ( $D^3 \cup_g S^3 \equiv D^3$ ) does not change topology of a disk:  $R \times S^3 + D^3 \cup_g S^3 = R^3 + S^3$ , but rather corresponds to a process of producing a disjoint union of  $R^3$  and  $S^3$ . This “cupping” or “plumbing” operation produces a complete manifold  $R^3$  (or  $R^3 + S^3$ ). The procedure described above is performed on the constant time initial data three-geometry. One would like to find an instanton solution corresponding to this decay mode. The simplest possible way to obtain such an instanton is to match a Euclidean black hole solution to a Friedmann universe on the constant time hypersurface. However, there are no vacuum solutions corresponding to this “match”. Consider a four manifold  $\mathcal{M}_F$  of topology  $S^3 \times R$  with a minimal  $S^3$ . Cut a wormhole  $\mathcal{M}_F$  in half on the minimal  $S^3$ . Next cut this minimal  $S^3$ , a constant “time” slice of a wormhole geometry, along an equator  $S^2$  obtaining thereby a disk  $D^3$ . A disk  $D^3$  is topologically the same as a disk  $D^3$  and a sphere  $S^3$  glued together:  $D^3 \equiv D^3 \cup_g S^3$ . Now glue the constant time slice of the Euclidean Schwarzschild spacetime (ES) along the “horizon”  $S^2$  to an equator of the minimal  $S^3$  of the half-wormhole. This operation defines the manifold of an instanton which mediates the decay of a black hole to a closed Friedmann universe and an asymptotically flat spacetime without a hole. The hybrid four-manifold obtained this way is the ES on the one side of the hypersurface of a constant time and the Friedmann universe on the other side.

Now we have to find a solution to the Euclidean equations of motion corresponding to the process of “pinching off” a small closed universe. Indeed, one can find such an instanton solution for the Kalb-Ramond two-form  $B$  coupled to gravity. There are other ways to obtain such an instanton, but from now on we focus our attention on this simple model. We simply observe that the Bekenstein result [14] which says that a static black hole does not have scalar (spin 0) hair can be easily extended to the Euclidean instanton case. The instanton solution in question is simply the ES on one part of an instanton manifold and a four-dimensional



wormhole on the other part of a complete instanton manifold. One simply requires smooth matching conditions on the metric and the second fundamental form of a matching surface. Indeed, one can show that such a smooth matching of the ES and the Friedmann universe does exist. In any case one need require only that the action of an instanton is finite.

The Euclidean Schwarzschild solution is known to possess the amazing property of having one normalizable negative mode in the “graviton” sector. What does this mean physically? Gross et al. [13] interpreted this fact as corresponding to a semiclassical nonperturbative instability of flat Minkowski spacetime in a thermal bath due to the nucleation of black holes (in contrast to the perturbative Jeans instability; see also Ref. (17) for the another point of view). Mathematically, this means that the ES instanton is only a local extremum of the Euclidean Einstein-Hilbert action. The possibility of the “phase transition” of a black hole in the “box” (in the “heat bath”) with a possible topology change was suggested in the different context in Ref. (17).

The four-dimensional wormhole (the Einstein-Rosen bridge) simply has the Euclidean Friedmann metric which is the solution of the coupled axion and Einstein equations. One can show that this solution is absolutely stable (in analogy to the real time Friedmann universe which is stable). This is implied by the positive action theorem. In the linear approximation, one finds no negative modes in the spectrum of deformations of the wormhole. On the other hand the half-wormhole does have negative modes in its spectrum of deformations. Observe that the time-symmetric initial value data corresponding to the 5-dimensional static Tangherlini black hole also describes a wormhole. However, this wormhole cannot be smoothly matched to the ES instanton. One of the interesting properties of this wormhole, which deserves a separate note, is the fact that the asymptotically Euclidean (AE) gravitational instanton with the topology  $R^4 \cup_g \mathcal{M}$  and zero Ricci scalar  $R$  has the Euclidean action greater or equal to the action of the Tangherlini wormhole. Indeed, one can show that the action of the AE gravitational instanton whose manifold is  $R^4$  outside a bounding  $S^3$  is not less than that of the Tangherlini in-

stanton, with the equality achieved only when the metric is that of the Tangherlini wormhole (Mazur 1987, unpublished).

The instanton solution described above mediates the decay of a black hole into a small closed universe “pinching off” from our “large” Universe containing a black hole. This instanton must correspond to a local extremum of the Euclidean action. This means that there exist negative modes in the spectrum of fluctuations around this instanton. Indeed, a wormhole cut in half and the Euclidean Schwarzschild do have negative modes. Therefore, the “matched” instanton solution corresponding to topology change  $S^2 \times R + D^3$  to  $S^3 + R^3$ , and from  $R^2 \times S^2$  to  $R \times S^3 + R^4$ , has negative modes in the “graviton” sector. In fact, there is only one normalizable negative mode around this instanton [15].

It is sufficient to present here the asymptotic form (as  $\tau \rightarrow -\infty$ ) of the metric on the wormhole cut in half

$$ds^2 = d\tau^2 + a^2(\tau)d\Omega_3^2,$$

$$a^2(\tau) \cong \tau^2(1 + 2R^4/3\tau^4). \quad (2)$$

Indeed, one shows that the exact solution to the coupled Einstein and the axion  $B$ -field equations can be given in the parametric form

$$a(\eta) = R(\cosh 2\eta)^{1/2}$$

$$\tau = \int d\eta a(\eta), \quad (3)$$

and  $dB = g(\tau)\epsilon$ , where  $\epsilon$  is the volume 3-form on  $S^3$ . From the axion equations of motion:  $d * H = 0$ ,  $H = dB$ , one finds

$$g = q/2\pi^2 f^2 a^3(\tau), \quad (4)$$

where  $q$  is an integer global axion charge. Here  $d\Omega_3^2$  is the metric on a unit round sphere  $S^3$  and  $R$  is the radius of the minimal  $S^3$  which equals the Schwarzschild

radius. Indeed, this radius  $R$  is determined by the condition that the Euclidean Friedmann solution matches the Euclidean Schwarzschild solution.  $R$  depends on the axion coupling  $f$  and the global charge  $q$ . We will not need the details of this relation here (this will be discussed elsewhere [15]). Anyway it is a rather trivial exercise. One finds the Euclidean action of the wormhole:  $I_W = 0$ , whereas the action of the ES instanton is:  $I_S = \pi M_p^2 R^2$ . The total action of the instanton is

$$I = I_S + I_W = \pi M_p^2 R^2, \quad (5)$$

because the action of the wormhole is zero. In terms of the black hole mass, the action is  $I = 4\pi M^2/M_p^2$ , where we have used  $R = 2MM_p^{-2}$ .

Now we can estimate a decay rate of a black hole with a mass  $M$ . Indeed, in the dilute instanton gas approximation one finds the decay rate per unit (Planck) volume [19]

$$\Gamma = A \exp(-I), \quad (6)$$

where  $A$  is the imaginary part of the functional determinant of the small fluctuation operator around an instanton. For the case under consideration, this is a determinant of the Lichnerowicz laplacian  $\Delta_L h_{ab} = -\nabla^2 h_{ab} + 2R_a{}^{cd} h_{cd} - 2R_{c(a} h_{b)}^c$  (in the more careful analysis one would also include the contribution from the axion field fluctuations). For our purposes we need only the order of magnitude estimate of  $A$ ,  $A \cong O(1)M_p^5 M^{-1}$ . The decay rate of a black hole thus equals

$$\Gamma \cong O(1)M_p^5 M^{-1} \exp(-4\pi M^2/M_p^2). \quad (7)$$

For a large black hole the decay rate due to this nonperturbative instability is extremely small and the decay time is much larger than the age of the Universe. However, for the mass of a small black hole,  $M = M_p$  the decay rate per unit spacetime volume is only of order  $10^{-6}$ . The spontaneous decay rate might indeed be small but the decay stimulated by the environment might be much higher.

Indeed, this is what we may expect to happen in the very early post-Planckian Universe. The mechanism described in this Letter might explain the absence of primordial black holes. Indeed, it seems that unlike the monopole problem the primordial black hole problem is easier to resolve because black holes are unstable, while monopoles are stable.

It should be noticed that the decay rate of a black hole due to the nonperturbative instability is proportional to  $\exp(-S_{bh})$ , where  $S_{bh}$  is the Bekenstein-Hawking entropy. This seems to indicate that the Bekenstein-Hawking entropy is the measure of the ability of a black hole to disappear from our Universe in the one quantum jump, with the simultaneous production of particles carrying away its total energy. Indeed, the naive estimate of a number of microstates which correspond to a given black hole of the mass  $M$  is  $N \cong \Gamma^{-1}$ . The microcanonical entropy for such a state is

$$S_{bh} = \ln N \cong -\ln \Gamma = 4\pi M^2 M_p^{-2}. \quad (8)$$

Zel'dovich has shown long time ago (in 1962, see Ref.(5)), that any number of baryons with any entropy can be brought into a configuration with an arbitrarily small rest mass. This is because the gravitational mass defect can almost completely compensate the total energy of the matter configuration. This implies that any material object is in fact metastable, or unstable against the decay mode with the transition to the more condensed state with an excess of energy being radiated away to infinity in the form of radiation. The apparent stability of matter is just a reflection of the fact that there exists an enormous energy barrier through which the system must tunnel to the superdense state. But the same principle also seems to apply to black holes. The Zel'dovich mechanism through which a number of baryons disappears from our Universe into a Closed World with the simultaneous production of particles with a zero net baryon number and the same total energy can proceed in two different channels. One of them is a formation of a wormhole

through which baryons disappear to another Universe, and the other is through the formation of a black hole which eventually decays via the mechanism described in this Letter. Both processes lead to the baryon number nonconservation.

In order to demonstrate the idea of the nonperturbative instability of black holes in QG it was sufficient to find only one decay mode of a black hole. The decay mechanism which becomes dominant at the Planck scale is due to topology change. There are other decay modes for which we have not constructed instantons. It is the total decay rate of all these “channels” which will give the correct decay time for a small black hole. Does this mean that one has to know all the decay channels to different topologies in order to calculate the total width of a quantum black hole state? One would expect that there will exist dominant channels for the decay such as the one we have demonstrated above. Evidently it is not clear at present to what degree one can trust the semiclassical calculations like those presented above. One has to bear in mind that the proper calculation of the decay rate  $\Gamma$  should be done in the hamiltonian formulation, i.e., using the WDW equation for the minisuperspace incorporating the properly matched Friedmann and Tolman metrics. The Euclidean approach to the tunnelling process described above (i.e., a black hole decay) could only be justified after the correct hamiltonian calculation was done (see, e.g., Ref.(15)). This approach will be described elsewhere [15]. In any case in this Letter we have presented not only the qualitative picture of the nonperturbative quantum gravitational decay of black holes but also a quantitative estimate of the decay rate for such a process. A more detailed discussion of the result presented here will be given in a forthcoming paper<sup>15</sup> where the Wheeler-De Witt equation for “isolated” gravitating objects is derived and applied to the quantum mechanics of black holes.

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Two very nice papers (Refs. 3,4), where the back-reaction effects are taken care of in the WKB approximation. Direct relation to tunneling is also quite clear. However, this work is really important because it gives the clear derivation of the nonlinear dispersion relation for quanta emitted by a black hole in the s-wave. This work also leads to the new way of calculating the very high-energy tail of the black hole decay in the D-brane context (Ref. 6 below).

5. P. Kraus, “Nonthermal Aspects of Black Hole Radiance”, Princeton University Ph.D. 1995 Thesis, 101 pp., gr-qc/9508007.
6. E. Keski-Vakkuri and P. Kraus, “Microcanonical D-Branes and Back Reaction”, Nucl. Phys. **B491**, 249-262 (1997), hep-th/9610045.
6. S. Massar and R. Parentani, “Gravitational Instanton for Black Hole radiation”, Phys. Rev. Lett. **78**, 3810-3813 (1997), gr-qc/9701015.

I have learnt from this paper, (Ref. 6), that the formula for the decay rate of a black hole due to the nonperturbative instability in the instanton approximation which was first derived in 1988 (in my paper presented above),  $\Gamma \sim e^{-S_{bh}} = e^{-\frac{1}{4}A_{bh}}$ , was also derived much later in Refs. 5,6 quoted in their

paper. I am not aware of earlier references which would relate the black hole entropy to the decay rate of a black hole due to tunneling. The exhaustive list of references to this subject can be found in the 1995 Ph.D. Thesis of P. Kraus (Ref. 5 above).